Optimal management of a wind power plant with storage capacity

Jérôme Collet[1] Olivier Féron[1] Peter Tankov[2]

[1]EDF Lab, Palaiseau

[2]ENSAE ParisTech, Palaiseau

September 13, 2019

Introduction

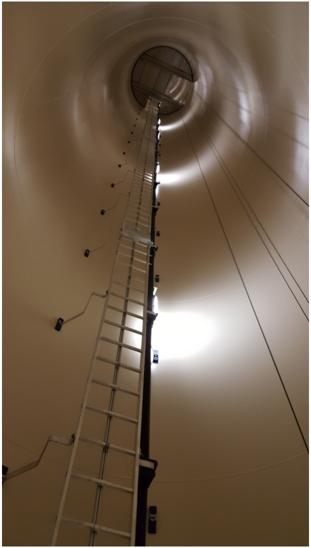
- Consider a wind producer who no longer has access to guaranteed purchase scheme and must sell the production in spot / intraday market.
- She can use battery storage capacity to smooth the variations of wind power production, and exploit intertemporal arbitrages in the day-ahead market.
- The aim is to maximize the expected gain of selling the energy produced during a 24-hour period.
- The dynamic strategy is constrained by finite battery capacity.

Introduction

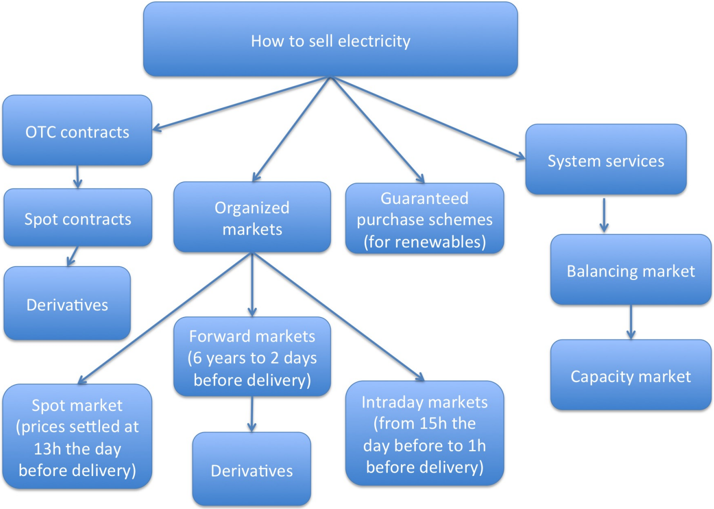
Australia has determined that Tesla 100MW battery system saved $40 million in 20 months with a cost of only $66 million.

Teslas 100MW/129MWh Powerpack project in South Australia provides the same grid services as peaker plants, but cheaper, quicker, and with zero-emissions.

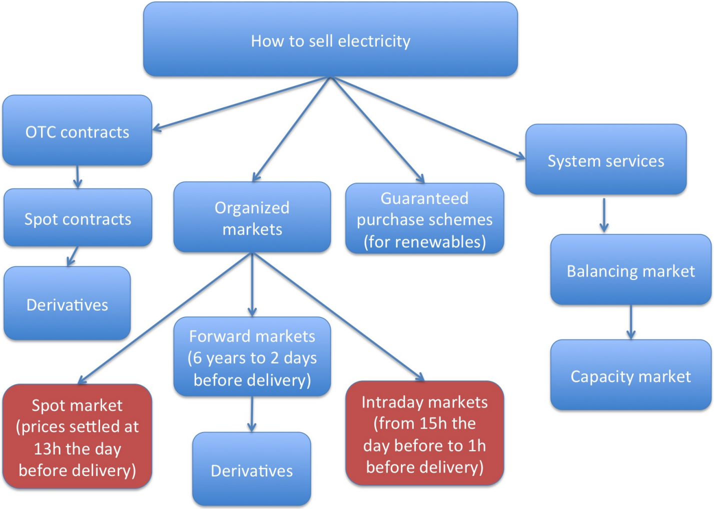
Introduction



How to sell electricity

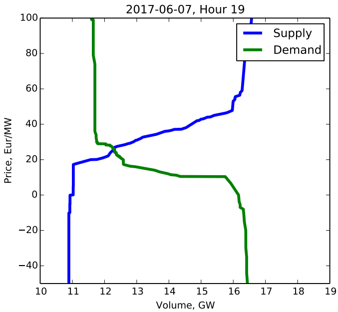


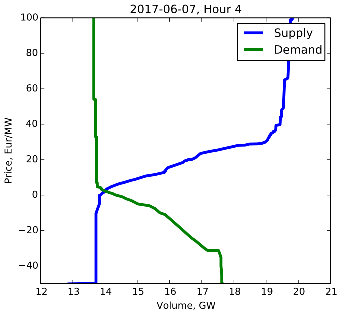
How to sell electricity



The spot (day-ahead) market

- One of the main trading venues for electricity is the day-ahead market (EPEX Spot in France/Germany).
- In this market trading happens only once: participants submit bids for specific hours of blocks of the next day until 12:00, then at 12:55 the price is fixed and market clears.



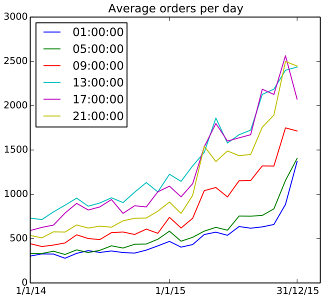


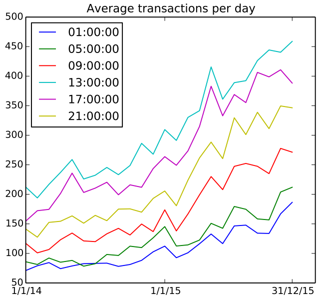
The intraday market

The intraday market opens at 15h and allows continuous trading for each hour/quarter-hour of the next day, up to 30 minutes before delivery.

Every delivery hour of every day corresponds to a different product: the life time of a single product is from 9 to 32 hours.

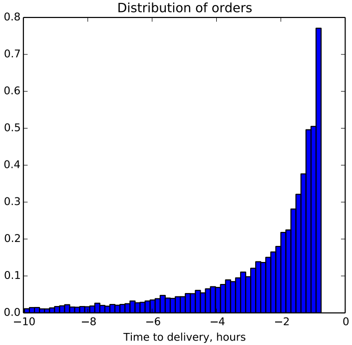
Market liquidity is improving but remains relatively low.





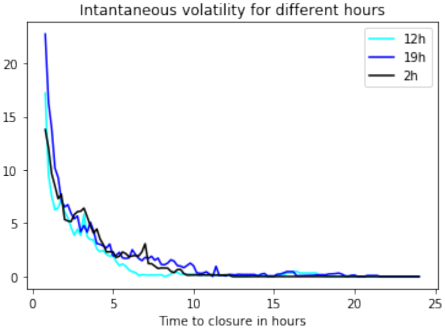
Intraday market liquidity patterns

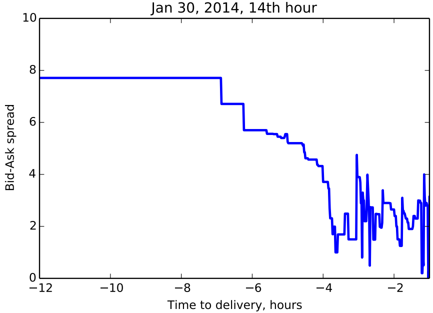
Liquidity only appears a few hours before delivery.



Distribution of orders/transactions as function of time to delivery for all contracts expiring in February 2015.

Bid-ask spread and volatility





Left: (Normal) volatility averaged over all days of February 2014 (kernel estimator, source: L. Tinsi). Right: bid-ask spread evolution on a typical day.

Strategy of the producer

- The producer makes bids in the day-ahead electricity market for the following day;
- This bid is based on an imperfect day-ahead forecast of the renewable production;
- He/ she may adjust the position for every hour once in the intraday market, one hour before delivery;
- At the time of adjustment, the production is known precisely.

Strategy of the producer

- The producer makes a bid in the spot (day-ahead) market at time $t = 0$, by making an engagement to deliver the amount $\overline{P}_k$ of electricity during the delivery period $[T_k + \delta, T_{k+1} + \delta]$ for each $k = 1, \dots, N$.
- These deliveries will be paid at the spot market price denoted by $F(0, T_k + \delta)$, $k = 1, \dots, N$.
- At each time T_k, the producer knows the amount of power, which will be generated during the delivery period $[T_k + \delta, T_{k+1} + \delta]$, and must decide how much power to buy/sell in the intraday market, and how much power to withdraw from / inject into the battery
- Injections/withdrawals must be balanced by production and market purchases.

Notation

Q_k	Energy stored in the battery at the beginning of delivery period $k+1$ (at time $T_{k+1}+\delta$)
Q_{min}	Minimal energy stored in the battery at all times
Q_{max}	Maximal battery capacity
p_k	Energy purchased in the intraday market during k-th delivery period $[T_k+\delta, T_{k+1}+\delta]$
P_k	Energy produced during k-th delivery period
$P(0, T_k)$,	Forecast at time 0 of energy production during k-th delivery period
$\overline{P}_k$	Energy delivered during k-th delivery period according to the engagements taken in the spot market
F_k	Intraday market price at time T_k for k-th delivery period
$F(0, T_k)$	Spot market price for k-th delivery period

Formulation of the optimization problem

The total gain from trading of the wind power producer is given by

$$G = \sum_{k=1}^{N} (\overline{P}_k F(0, T_k) - F_k(p_k + \alpha|p_k|)),$$

where the term $\alpha|p_k|$ models the bid-ask spread in the intraday market. The aim of the producer is to maximize the expected gain under storage constraint

$$Q_k \in [Q_{min}, Q_{max}], \quad Q_k = Q_{k-1} + P_k - \overline{P}_k + p_k.$$

The optimization problem of the producer thus writes:

$$\max_{\overline{P}_1,\dots,\overline{P}_N, p_1,\dots,p_N} \left\{ \sum_{k=1}^{N} \overline{P}_k F(0, T_k) - \mathbb{E}\left[\sum_{k=1}^{N} F_k(p_k + \alpha|p_k|) \right] \right\},$$

where $\overline{P}_1, \dots, \overline{P}_N$ are constants (determined at time 0), and $(p_k)_{1 \le k \le N}$ is a dynamic strategy of trading in the intraday market.

Modeling the intraday price process

We model the intraday price as a perturbation around the day-ahead price.

$$F_t = F(0,t) + \bar{\alpha}(t)(\overline{X}_t + \bar{\mu}(t)),$$

where $\overline{X}_t = \bar{\sigma}\int_0^t e^{-\lambda(t-s)}dB_s$ is a centered Gaussian factor process (Ornstein-Uhlenbeck), where B is a Brownian motion.

Here we recall that F_t is the “last” intraday price and $F(0,t)$ is the day-ahead price.

$\bar{\alpha}$ and $\bar{\mu}$ model the daily seasonality of the intraday price process.

The day-ahead price is considered as given: the optimization is performed within a day, when the day-ahead price is known

Modeling the intraday price curve: risk-neutral dynamics

For a more precise description, use a multifactor model for the last intraday price

$$F_t = F(0,t) + \bar{\alpha}(t) \sum_{j=1}^{M'} Y_t^j,$$

where $(Y^j)_{j=1}^{M'}$ are independent Ornstein-Uhlenbeck processes:

$$dY_t^j = -\bar{\lambda}^j Y_t^j dt + \bar{\sigma}^j d\widehat{B}_t^j,$$

where, $(\widehat{B}^j)_{j=1}^{M'}$ are Brownian motions under the risk-neutral measure $\mathbb{Q}$.

Intraday prices at other times are computed by taking risk-neutral expectation:

$$F(t,T) = \mathbb{E}[F_T|\mathcal{F}_t] = F(0,T) + \sum_{j=1}^{M'} \bar{\alpha}(T) e^{-\bar{\lambda}_j (T-t)} Y_t^j.$$

Modeling the intraday price curve: real-world dynamics

To obtain the real-world dynamics, change probability

$$\frac{d\mathbb{P}}{d\mathbb{Q}}\Big|_{\mathcal{F}_T} = \exp\left(-\int_0^T \phi_t dB_t - \frac{1}{2}\int_0^T \phi_t^2 dt\right),$$

where ϕ is deterministic. The process

$$B_t = \widehat{B}_t + \int_0^t \phi_s ds$$

is then a Brownian motion under $\mathbb{P}$ and

$$Y_t^j = \bar{\sigma}^j \int_0^t e^{-\lambda_j(t-s)} dB_s^j + \bar{\sigma}^j \int_0^t e^{-\lambda_j(t-s)} \phi_s^j ds := \bar{\sigma}^j \int_0^t e^{-\lambda_j(t-s)} dB_s^j + \bar{\mu}^j(t),$$

so that

$$F_t = F(0,t) + \bar{\alpha}(t) \sum_{j=1}^{M'} (\overline{Y}_t^j + \bar{\mu}^j(t)),$$

where $\overline{Y}_t^j = \bar{\sigma}^j \int_0^t e^{-\lambda_j(t-s)} dB_s^j$ is a centered Gaussian factor process.

Modeling the production process

Similarly, we model the realized production as a perturbation around the day-ahead production forecast.

$$P_t = P(0,t) + \alpha(t)(1 + \gamma P(0,t)^\delta)(X_t + \mu(t)),$$

where $X_t = \sigma \int_0^t e^{-\lambda(t-s)} dW_s$ is a centered Gaussian factor (Ornstein-Uhlenbeck) which may be correlated with the price noise $\overline{X}$.

α and μ^j model the daily seasonality of the production process and $P(0,t)$ is the forecast at the gate closure time of the intraday market.

Model allows for negative production values, but such values may be possible in practice when the wind speed is very low.

The extra factor $(1 + \gamma P(0,t)^\delta$ translates the fact that the forecast errors are higher when the production is higher.

Modeling the dynamics of the forecast curve

Multifactor model for the production process

$$P_t = P(0,t) + \alpha(t)(1+\gamma P(0,t)^\delta)\sum_{j=1}^M (X_t^j + \mu^j(t)),$$

where $(X^j)_{j=1}^M$ are independent Ornstein-Uhlenbeck processes:

$$dX_t^j = -\lambda_j X_t^j dt + \sigma_j dW_t^j.$$

The forecast processes at other times are given by

$$\begin{aligned} P(t,T) &= \mathbb{E}^{\mathbb{P}}[P_T|\mathcal{F}_t] \\ &= P(0,T) + \sum_{j=1}^M \left\{\alpha(T)(1+\gamma P(0,T)^\delta)e^{-\lambda_j(T-t)}(X_t^j + \mu^j(t))\right\}. \end{aligned}$$

Model calibration

- We focus on the production model, in the one-factor case
- Assume that the agent observes L indepenent realizations of the forecast $(P^l(0,T_k))_{k=1,\dots,N}^{l=1,\dots,L}$ and the production process $(P_k^l)_{1\leq k\leq N}^{l=1,\dots,L}$
- Each realization corresponds to a single production day in the past
- Introduce the forecast error process $(Z_k^l)_{1\leq k\leq N}^{1\leq l\leq L}$, where $Z_k^l = P_k^l - P^l(0,T_k)$.
- Z_k^l is a Gaussian vector and the log-likelihood of $(Z_k^l)_{k=1,\dots,N}^{l=1,\dots,L}$ is

$$l(\alpha,\mu,\lambda,\gamma,\delta) = -\frac{L}{2}\log(\det\Omega(\lambda)) - L\sum_{j=1}^{N}\log\alpha_j - \sum_l\sum_j \log(1+\gamma P^l(0,T_j)^\delta)$$

$$-\frac{1}{2}\sum_{l=1}^{L}\left(\frac{Z_{\gamma,\delta}^l}{\alpha}-\mu\right)^\top \Omega^{-1}(\lambda)\left(\frac{Z_{\gamma,\delta}^l}{\alpha}-\mu\right),$$

where $\alpha_k := \alpha(T_k)$, $\mu_k = \mu(T_k)$ and $Z_{\gamma,\delta}^l = Z^l/(1+\gamma P^l(0,T)^\delta)$.

Maximizing the likelihood function

Differentiating with respect to μ, we get an explicit expression

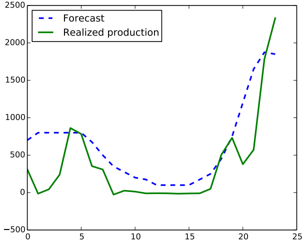
$$\mu = \frac{1}{L\alpha}\sum_{l=1}^{L} Z^l_{\gamma,\delta} := \frac{1}{\alpha}\overline{Z}_{\gamma,\delta},$$

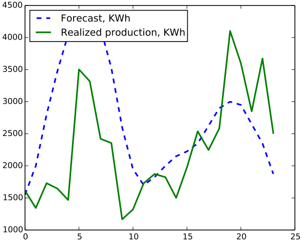
and a simplified log-likelihood

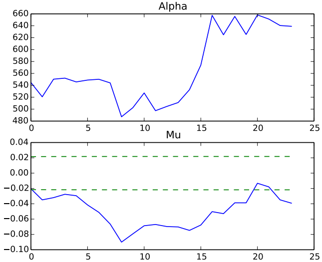
$$l(\alpha,\lambda,\gamma,\delta) = -\frac{L}{2}\log(\det\Omega(\lambda)) - L\sum_{j=1}^{N}\log\alpha_j - \sum_l\sum_j \log(1+\gamma P^l(0,T_j)^\delta)$$
$$-\frac{1}{2}\sum_{l=1}^{L}\left(\frac{Z^l_{\gamma,\delta}}{\alpha} - \frac{\overline{Z}_{\gamma,\delta}}{\alpha}\right)^\top \Omega^{-1}(\lambda)\left(\frac{Z^l_{\gamma,\delta}}{\alpha} - \frac{\overline{Z}_{\gamma,\delta}}{\alpha}\right).$$

which we minimize with a numerical algorithm.

Numerical illustration: forecast







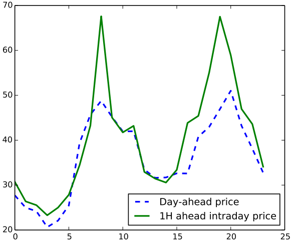
Top: Day-ahead forecast and realized production on Sep 8, 2014 (left) and Oct 19, 2014 (right).

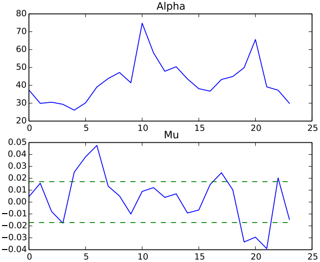
Bottom: estimation of the model for realized production.

Data: production and forecast (at 12h the day before) for a wind parc in France, Jan 1st 2012 – Dec 31st 2014.

Length of mean reversion: about 6.2 hours.

Numerical illustration: price





Top: Day-ahead price and last intraday price on Sep 8, 2014 (left) and Oct 19, 2014 (right).

Bottom: Estimation of the intraday price model. Data:

EPEX Spot day-ahead and intraday price,

Germany-Austria region, Jan 1st 2014 – Dec 31st 2014.

Length of mean reversion: about 2.5 hours.

Solving the optimization problem: dynamic programming

Recall that we need to solve

$$\max_{\overline{P}_1,\dots,\overline{P}_N,p_1,\dots,p_N} \left\{ \sum_{k=1}^{N} \overline{P}_k F(0,T_k) - \mathbb{E}\left[\sum_{k=1}^{N} F_k(p_k + \alpha |p_k|) \right] \right\},$$

where $\overline{P}_1,\dots,\overline{P}_N$ are determined at time 0 and $(p_k)_{1\le k\le N}$ are determined dynamically

We solve this problem by discrete-time dynamic programming (backward induction)

State variables are: battery charge state $(Q_k)_{1\le k\le N}$ and the factor processes for the wind production $(X_{T_k})_{1\le k\le N}$ and the intraday market price $(Y_{T_k})_{1\le k\le N}$. We consider all processes in discrete time.

Solving the optimization problem: dynamic programming

Define the value function

$$v_k(q, x, y) = \min_{p_k,\dots,p_N, Q_{k-1}=q} \mathbb{E}^{T_k,x,y}\left[\sum_{n=k}^{N} F_n(p_n + \alpha|p_n|)\right],$$

where $\mathbb{E}^{T_k,x,y}$ means that we start the factor processes at time T_k from values x and y.

The original optimization problem then writes

$$\max_{\overline{P}_1,\dots,\overline{P}_N} \left\{ \sum_{k=1}^{N} \overline{P}_k F(0, T_k) - \mathbb{E}^{0,X_0,Y_0}[v_1(Q_0, X_{T_1}, Y_{T_1})] \right\}.$$

Solving the optimization problem: dynamic programming

The dynamic programming principle for the value function writes

$$v_k(q,x,y) = \min_{p_k : q+P_k-\overline{P}_k+p_k \in [Q_{min},Q_{max}]} \{\phi_k(y)(p_k+\alpha|p_k|) + \mathbb{E}^{T_k,x,y}[v_{k+1}(q+\pi_k(x)-\overline{P}_k+p_k, X_{T_{k+1}}, Y_{T_{k+1}})]\}$$

with the terminal condition

$$v_N = v_N(q,x,y) = \min_{p_N : q+P_N-\overline{P}_N+p_N \in [Q_{min},Q_{max}]} \phi_N(p_N+\alpha|p_N|),$$

where

$$\phi_k(y) = F(0,T_k) + \bar{\alpha}(T_k)(y+\bar{\mu}(T_k)),$$
$$\pi_k(x) = P(0,T_k) + \alpha(T_k)(1+\gamma P(0,T_k)^{\delta})(x+\mu(T_k)).$$

Solving the optimization problem: battery state

Discretize the state of charge of the battery on a uniform grid $Q_{min} = q_1 < \cdots < q_J = Q_{max}$.

$\Rightarrow$ The control p_k takes a finite number of values.

We denote $v_k(q_j, \dots)$ by v_k^j. Then,

$$v_k^j(x, y) = \min_{i=1,\dots,J} \{\phi_k(y)\eta(q_i - q_j + \overline{P}_k - \pi_k(x)) + \mathbb{E}^{T_k,x,y}[v_{k+1}^i(X_{T_{k+1}}, Y_{T_{k+1}})\},$$

where $\eta(p) = p + \alpha|p|$.

Solving the optimization problem: quantization

Let $\mathbb{P}_k$ be the unconditional distribution of $Z := (X_{T_k}, Y_{T_k})$. For every $k = 1, \dots, N$, define a grid of size N_q by solving

$$\min_{\hat{Z}} \mathbb{E}^{\mathbb{P}_k}[(Z - \hat{Z})^2],$$

where the minimum is taken over all random vectors supported by N_q points.

The solution (optimal Voronoi quantization) is obtained by nearest-neighbor projection of Z on a set of N_q points.

We denote these points by $\hat{z}_1^k, \dots, \hat{z}_{N_q}^k$ with $\hat{z}_j^k := (\hat{x}_j^k, \hat{y}_j^k)$, the associated Voronoi cells by $C_1^k, \dots, C_{N_q}^k$ and the associated probabilities by $\hat{p}_1^k, \dots, \hat{p}_{N_q}^k$.

The points $\hat{z}$ can be computed using the K-means algorithm or downloaded from `quantize.maths-fi.com`.

Solving the optimization problem: quantization

Next, we replace the continuous process with a Markov chain $(\hat{Z}_k)_{0\leq k\leq N}$ with N_q states. The transition probabilities of the chain are defined by

$$\hat{\pi}_i^0 = \mathbb{P}[\hat{Z}_1 = \hat{z}_i^1] = \hat{\pi}_i^1$$

$$\text{and} \quad \hat{\pi}_{ij}^k = \mathbb{P}[\hat{Z}_{k+1} = \hat{z}_j^{k+1} | \hat{Z}_k = \hat{z}_i^k] = \mathbb{P}[Z_{T_{k+1}} \in C_j^{k+1} | Z_{T_k} \in C_i^k].$$

These transition probabilities are evaluated by Monte Carlo.

The value function can then be computed on the quantization grid using the following formula:

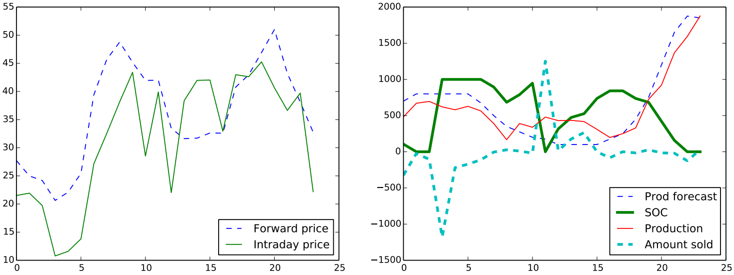
$$v_k^j(\hat{z}_m^k) = \min_{i=1,\dots,J}\{\phi_k(\hat{y}_m^k)\eta(q_i - q_j + \overline{P}_k - \pi_k(\hat{x}_m^k)) + \sum_{l=1}^{N_q} \hat{\pi}_{ml}^k v_{k+1}^i(z_l^{k+1})\}.$$

Solving the optimization problem

- The value function and the optimal strategies are computed numerically by dynamic programming
- We start by discretizing the state of charge of the battery, introducing a uniform grid $Q_{min} = q_1 < \cdots < q_J = Q_{max}$.
- The second step is to replace the discrete-time Ornstein-Uhlenbeck processes $(\overline{X}, X)$ with a finite-state Markov chain. This will be achieved using the method of *optimal quantization*.
- The value function can then be computed on the quantization grid

Numerical illustration: sample strategies

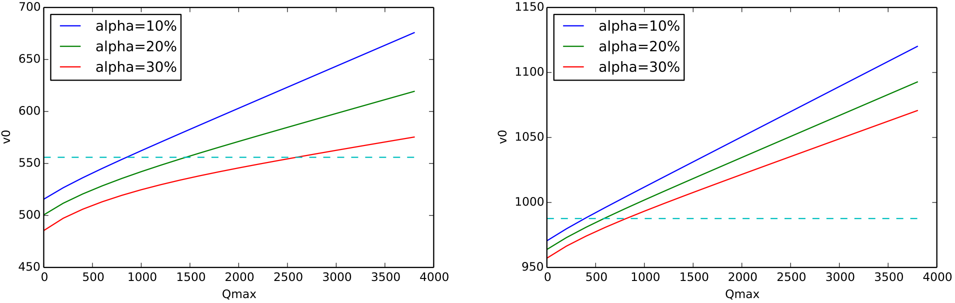
The spot market engagements $\overline{P}_k$ have been taken equal to production forecasts for the corresponding hour: $\overline{P}_k = P(0, T_k)$.



Sample evolution of the modelled quantities. In the left graph, prices are in Euros per MWh. In the right graph, all amounts are shown in KWh.

Numerical illustration: battery capacity

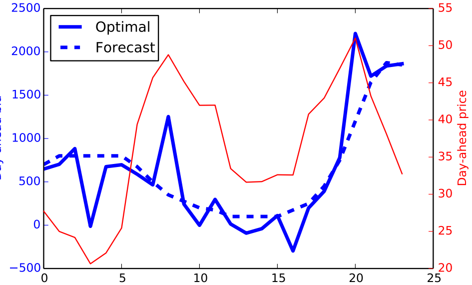
Maximum expected gain of the power producer for different values of the battery capacity Q_{max} and different values of α.

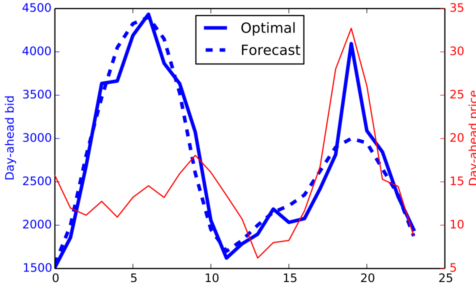


The dotted line shows the theoretical profit of the power producer if the power production were exactly equal to the day-ahead forecast. Left: 8 September 2014. Right: 19 October 2014.

Numerical illustration: optimal bids in the spot market

These are obtained by maximizing the value function of the producer with fixed bids $\overline{P}_k$, with respect to $\overline{P}_k$ with a numerical optimization algorithm (BFGS).

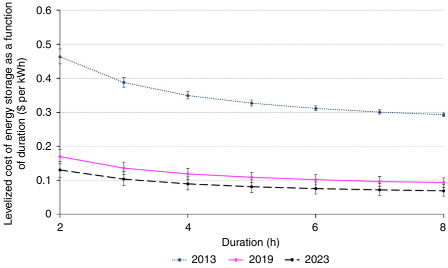




Left graph: 8 September 2014. Right graph: 19 October 2014.

Conclusion

- Methodology for optimal management of a wind plant - battery system based on stochastic control;
- Strategy is initialized daily: it allows to determine optimal day-ahead bids and then adjust intraday position as production becomes known.
- At present, cost of battery storage is around 10 cents per KWh produced: a 100 euro price differential is needed to implement arbitrage.
- Falling cost of batteries may boost profitability.



Source: Comello and Reichelstein, Nature Comm. (2019) 10:2038