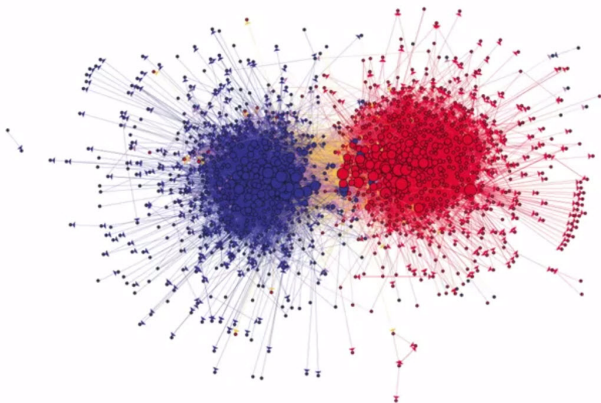


Statistical Network Analysis

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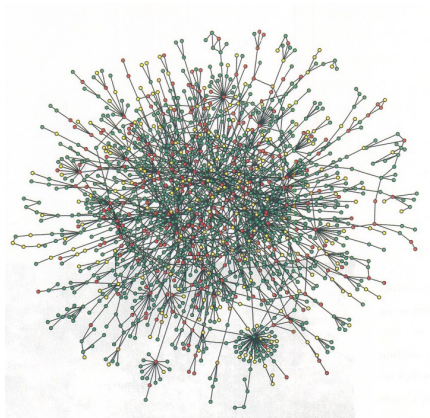
Social Political Blog Network



Problem

Bisection of the graph

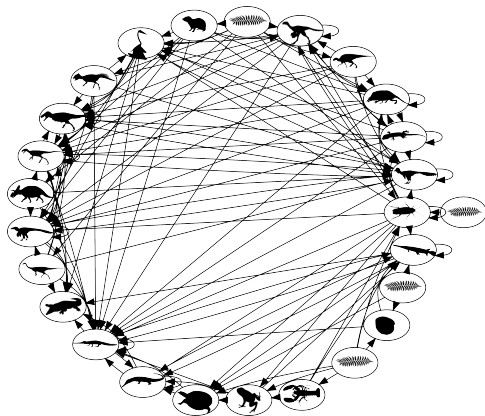
Yeast Protein-Protein Interaction



Problem

Find the hubs

Trophic Network



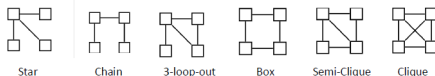
Problem

Find groups with strong connexions

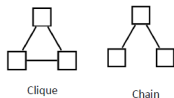
Other structures...

- Determine motifs :

Connected Motifs of size 4



Connected Motifs of size 3



Motifs = recurrent and statistically significant sub-graphs or patterns.

- Detection of anomalies (e.g. detection of hacking attacks)
- ...

A null Model: Erdős-Rényi

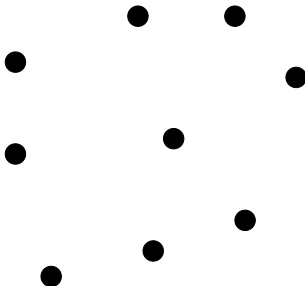
Erdős-Rényi random graph

$\mathbb{G}(n, p)$: $(\mathbf{A}_{ij} : i < j)$ are i.i.d. Bernoulli(p) and $\mathbf{A}_{ii} = 0$.

A null Model: Erdős-Rényi

Erdős-Rényi random graph

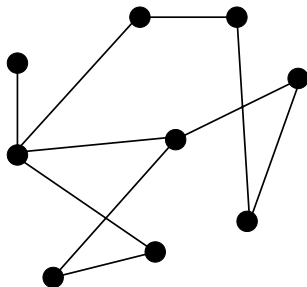
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A null Model: Erdős-Rényi

Erdős-Rényi random graph

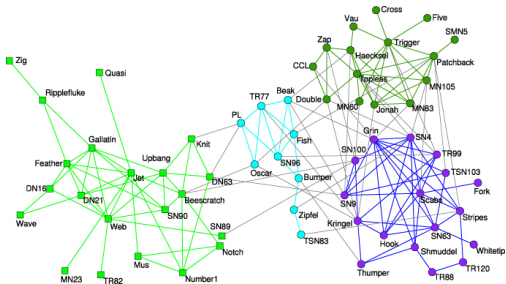
$\mathbb{G}(n, p)$: $(\mathbf{A}_{ij} : i < j)$ are i.i.d. Bernoulli(p) and $\mathbf{A}_{ii} = 0$.



Remark: $\text{deg}(i) \sim \text{Bin}(n - 1, p)$.

- if p is constant, the mean degree is proportional to n (**dense** case)
- if $p \approx c/n$, the mean degree is of constant order (**sparse** case)

Real world networks



- **heterogeneous** structure
- **clustering** nodes that play the same role in the graph
- **sparse**

Dolphin Social Network [Newman and Girvan(04)]